The University of Georgia

Mathematics Education
EMAT 4680/6680 Mathematics with Technology
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Exploration #4: The Centroid

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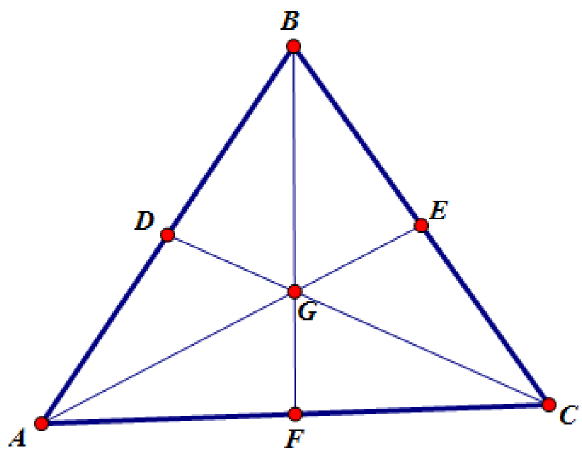
The Centroid (G) of a triangle is the common intersection of the three medians. A median of a triangle is the segment from a vertex to the midpoint of the opposite side.

Prove that the medians divide the triangle into six small triangles. Show that these triangles all have the same area.

Let's observe Δ ABC below:



Points D,E,F are midpoints of the three sides of Δ ABC.

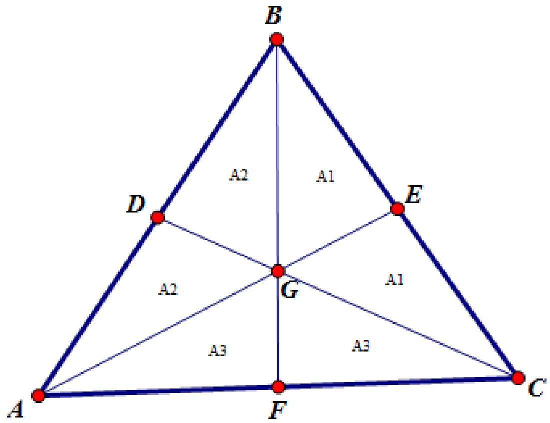
G is the centroid of ΔABC, which is the point where the three medians of a triangle intersect. A median is a line segment that connects a vertex of the triangle to the midpoint of the opposite side of that triangle. Segments AE, CD, and BF are all medians in the diagram above.

Let us first examine ΔBGC.

We know that line segment BE=EC because E is the midpoint of the line segment BC, which by definition is the middle point of a line segment.

Now let us look at ΔEGC and ΔBGE. We have already concluded that line segments BE and EC are congruent. In addition, the two triangles share the same base GE. So ΔEGC and ΔBGE must have the same area, because the area of a triangle is defined as ½ base*height, and the triangles share the same based of BE=CE and the same height of GE. Let us define the area of each of these triangles as A_1.

We can now apply the aforementioned to ΔAGB and ΔAGC.
We will define the area of triangles ΔAGD and ΔBGD as A_2.
We will define the area of triangles ΔAGF and ΔCGF as A_3.



Let us now examine two separate triangles: ΔAEB and ΔAEC.
We can use the areas of the 6 triangles in the diagram above.
The area of ΔAEB=$2A_2+A_1$. The area of ΔAEC=$2A_3+A_1$.

We can show that the area of ΔAEB and ΔAEC are equal. We know that segments BE and CE are equal because E is the midpoint of BC. Therefore, the triangles have equal bases. In addition, both triangles share segment AE which is the height of both triangles.

Therefore, we can conclude the following: Area of ΔAEB = Area of ΔAEC.
Therefore, $2A_2+A_1=2A_3+A_1$. We can simplify and conclude that $A_2=A_3$.

Let us now examine two separate triangles: ΔBFC and ΔBFA.
We can use the areas of the 6 triangles in the diagram above.
The area of ΔBFC=$2A_1+A_3$. The area of ΔBFA=$2A_2+A_3$.

We can show that the area of ΔBFC and ΔBFA are equal. We know that segments AF and FC are equal because F is the midpoint of AC. Therefore, the triangles have equal bases. In addition, both triangles share segment BF which is the height of both triangles.

Therefore, we can conclude the following: Area of ΔBFC = Area of ΔBFA.
Therefore, $2A_1+A_3=2A_2+A_3$. We can simplify and conclude that $A_1=A_2$.

We have just shown that $A_2=A_3$ and $A_1=A_2$, so therefore, $A_1=A_3$ and $A_1=A_2=A_3$.

Therefore, the areas of the six smaller triangles who all share the centroid of the triangles as a point, created from the three medians of the triangle, have the same area.